Enrollment No: ______________________ Exam Seat No: ______________________

C. U. SHAH UNIVERSITY
Summer Examination-2022

Subject Name: Mathematical Methods - I

Subject Code: 5SC03MAM1 **Branch: M.Sc. (Mathematics)**

Semester: 3 **Date: 22/04/2022** **Time: 02:30 To 05:30** **Marks: 70**

Instructions:

1. Use of Programmable calculator and any other electronic instrument is prohibited.
2. Instructions written on main answer book are strictly to be obeyed.
3. Draw neat diagrams and figures (if necessary) at right places.
4. Assume suitable data if needed.

SECTION – I

Q-1 **Attempt the Following questions** **(07)**

(a) State first shifting theorem for Laplace transforms. 02

(b) Find $L\left\{\frac{1}{\sqrt{\pi t}}\right\}$. 02

(c) Define: Direct delta function. 01

(d) $$L^{-1}\left\{\frac{\bar{f}(s)}{s}\right\} = \_\_\_\_\_.$$ 01

(e) $$Z(e^{an}) = \_\_\_.$$ 01

Q-2 **Attempt all questions** **(14)**

(a) Prove that $Z(\cosh n\theta) = \frac{z^2+2z\cosh\theta}{(z^2-2z\cosh\theta+1)}$. 05

(b) If $L\{f(t)\} = \bar{f}(s)$ then prove that $L\{t^n f(t)\} = (-1)^n \frac{d^n}{dx^n}\left[\bar{f}(s)\right]$. 05

(c) Find Laplace transform of the periodic function $f(t)$ with period $\frac{2\pi}{\omega}$. 04

Where, $f(t) = \begin{cases} \sin\omega t\,;\, 0 < t < \frac{\pi}{\omega} \\ 0;\ \frac{\pi}{\omega} < t < \frac{2\pi}{\omega} \end{cases}$

OR

Q-2 **Attempt all questions** **(14)**

(a) Find the Z transform and region of convergence of 05

$u(n) = \begin{cases} 4^n & for\ n < 0 \\ 2^n & for\ n \geq 0 \end{cases}$.

(b) Find $Z^{-1}\left\{\frac{2(z^2-5z+6.5)}{(z-2)(z-3)^2}\right\}, 2 < |z| < 3.$ 05

(c) Find $L\left\{\frac{1-\cos 2t}{t}\right\}$. 04

Q-3 **Attempt all questions** **(14)**

(a) Prove that $L\{\mathrm{erf}(\sqrt{x})\} = \frac{1}{s\sqrt{s+1}}$. 05

(b) If $f(t)$ is periodic function with period T then prove that 05

$$L\{f(t)\} = \frac{1}{1-e^{-sT}}\int_0^T e^{-st} f(t)dt.$$

(c) If $L^{-1}\left\{\frac{s}{(s^2+1)^2}\right\} = \frac{1}{2}t\sin t$, then find $L^{-1}\left\{\frac{32s}{(16s^2+1)^2}\right\}$. 04

OR

Q-3 **Attempt the Following questions** **(14)**

(a) State and prove Convolution theorem for Laplace transform. 05

(b) Find $L\left\{\int_0^t \frac{e^{-t}\sin t}{t}dt\right\}$. 05

(c) If $\{u_n\}$ be any discrete sequence and $Z\{u_n\} = U(z)$ then prove that 04
$(i) Z(a^{-n}u_n) = U(az)$ and $(ii)\ Z(a^n u_n) = U\left(\frac{z}{a}\right)$.

SECTION – II

Q-4 **Attempt the Following questions** **(07)**

(a) Define: Z-transform 02

(b) Check whether the function $f(x) = \begin{cases} 0\,; -2 < x < -1 \\ k;\quad -1 < x < 1 \\ 0\,;\quad 1 < x < 2 \end{cases}$ is even or odd? 02

(c) If F(λ) is Fourier transform of f(x) then prove that 02

$$\mathrm{F}\{\mathrm{f(x)}\cos \mathrm{ax}\} = \frac{1}{2}\{\mathrm{F}(\lambda-\mathrm{a}) + \mathrm{F}(\lambda+\mathrm{a})\}$$

(d) Define: Inverse Fourier transform. 01

Q-5 **Attempt all questions** **(14)**

(a) State and prove Parseval's formula for Fourier series. 05

(b) If $F(\lambda)$ is Fourier transform of $f(x)$ then prove that 05

$$F\{f(ax)\} = \frac{1}{a}F\left(\frac{\lambda}{a}\right)\ ; a \neq 0.$$

(c) Find Fourier sine series of period 4 for the function 04

$$f(x) = \begin{cases} 2x & ; 0 < x < 1 \\ 4-2x & ; 1 < x < 2 \end{cases}.$$

OR

Q-5		**Attempt all questions**	**(14)**
	(a)	Find the Fourier series of $f(x) = x^2$ in the interval $0 < x < 2$.	06
	(b)	Express $e^{-x}\cos x$ as a Fourier cosine integral and show that $e^{-x}\cos x = \frac{2}{\pi}\int_0^\infty \frac{(\lambda^2+2)}{\lambda^4+4}\cos\lambda x\ d\lambda$.	05
	(c)	If $F(\lambda)$ is Fourier transform of $f(x)$ then prove that $\mathcal{F}[f(ax)] = \frac{1}{a}F\left(\frac{\lambda}{a}\right), a \neq 0$.	03
Q-6		**Attempt all questions**	**(14)**
	(a)	Find Fourier integral representation of function $f(x) = \begin{cases} 1\ ; \|x\| < 1 \\ 0\ ; \|x\| > 1 \end{cases}$ and hence evaluate $\int_0^\infty \frac{\sin\lambda\cos\lambda x}{\lambda}\ d\lambda$.	07
	(b)	Find Fourier transform of $f(x) = e^{-a^2x^2}; a > 0$ and hence deduce that $F\left(e^{-\frac{x^2}{2}}\right) = e^{-\left(\frac{\lambda^2}{2}\right)}$.	07

OR

Q-6		**Attempt all Questions**	**(14)**
	(a)	Solve: $\frac{\partial y}{\partial t} = 2\left(\frac{\partial^2 y}{\partial x^2}\right)$,where $y(0,t) = y(5,t) = 0$ and $y(x,0) = 10\sin 4\pi x$.	09
	(b)	Find Fourier cosine transform of e^{-x^2}.	05